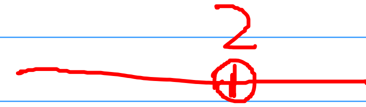
TD6 :

2020/12/09

G3.

Jiao HE

Ex6 : ② $f(x) = \dfrac{3x^2}{x-2}$. $D_f = \mathbb{R} \setminus \{2\}$.



$$f'(x) = 3 \times \frac{2x \times (x-2) - x^2 \times 1}{(x-2)^2} = 3 \times \frac{2x^2 - 4x - x^2}{(x-2)^2}$$

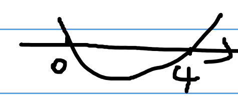
$$= 3 \times \frac{x^2 - 4x}{(x-2)^2}$$

$$f'(x) = \frac{6x \times (x-2) - 3x^2 \times 1}{(x-2)^2} = \frac{6x^2 - 12x - 3x^2}{(x-2)^2} = \frac{3x^2 - 12x}{(x-2)^2}.$$

$f'(x)$ a le signe du poly $x^2 - 4x = x(x-4)$, $(x-2)^2 > 0$ pour $x \in D_f$.

$f'(x) > 0$ pour $x \in]-\infty, 0] \cup [4, +\infty[$, donc $f(x) \uparrow$

$f'(x) < 0$ pour $x \in [0, 4]$, donc $f(x) \downarrow$



le tableau de variation.

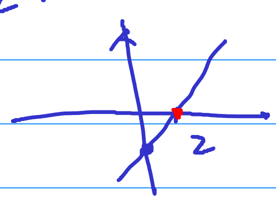
x	$-\infty$		0		2		4		$+\infty$
$f'(x)$		$+$	0	$-$	‖		0	$+$	
$f(x)$	$-\infty$	$\nearrow$	0	$\searrow$ $-\infty$	‖ $+\infty$	$\searrow$	24	$\nearrow$	$+\infty$

$f(0) = 0$ $\lim\limits_{x \to 2^+} f(x) = +\infty$ $f(4) = 24$ $\lim\limits_{x \to -\infty} f(x) = -\infty$

$\lim\limits_{x \to 2^-} f(x) = -\infty$ $\lim\limits_{x \to +\infty} f(x) = +\infty$

✓

$f(x) = \dfrac{3x^2}{x-2}$. sur $\mathbb{R} \setminus \{2\}$. $3x^2 \geq 0$ pour tout $x \in \mathbb{R}$.



$\lim\limits_{x \to 2^-} x - 2 = 0$ $\lim\limits_{x \to 2^+} x - 2 = 0$

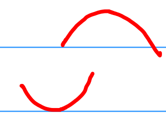
$\dfrac{1}{x-2}$ $\lim\limits_{x \to 2^-} \dfrac{1}{x-2} = -\infty$ $\lim\limits_{x \to 2^+} \dfrac{1}{x-2} = +\infty$.

$x - 2 < 0$ $x - 2 > 0$

Ex7 : ① $u = \ln(x) - e^{x-1}$ est définie pour $x > 0$.

rappelle : concavité $\Leftrightarrow f''(x) < 0$ graphe f

convexité $\Leftrightarrow f''(x) > 0$ f



montrer que u est strictement concave $\Leftrightarrow u'' < 0$.

$u'(x) = \frac{1}{x} - e^{x-1}$

$u''(x) = -1 \times x^{-2} - e^{x-1}$

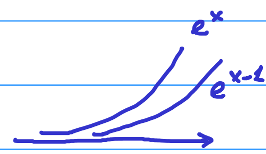
$= \underbrace{-\frac{1}{x^2}}_{<0} \underbrace{- e^{x-1}}_{<0} < 0$

$(\ln(x))' = \frac{1}{x}$

$(e^x)' = e^x$

$(e^{x-1})' = e^{x-1} \times (x-1)'$

$= e^{x-1}$



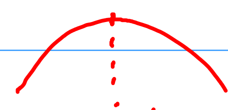
donc $u'' < 0$. c-à-d. u est strictement concave.

② $u'(x) = \frac{1}{x} - e^{x-1}$

$u'(1) = \frac{1}{1} - e^{1-1} = 1 - e^0 = 0$.

$\boxed{u'(x) = 0 \text{ en point } x = 1.}$

③ u est strictement concave. donc u admet un unique maximum global en $x = 1$.



TD7 :

Ex2. ① $P_D(q) = \frac{9}{q+1}$ $P_O(q) = q+1$, q : quantité , $q>0$.

p : prix .

à l'équilibre on a $P_D(q) = P_O(q)$

donc $\frac{9}{q+1} = q+1 \Rightarrow 9 = (q+1)^2$

$\Rightarrow q+1 = \cancel{-3}, +3$

$\Rightarrow q = 2$.

donc $q^* = 2$. $p^* = 2+1 = 3$

② le surplus du consommateur : $SC = \int_0^{q^*} P_D(q)\,dq - p^* q^*$ définition

le surplus du producteur : $SP = p^* q^* - \int_0^{q^*} P_O(q)\,dq$

$$SC = \int_0^2 \frac{9}{q+1}\,dq - 3\times 2 = 9\int_0^2 \frac{1}{q+1}\,dq - 6$$

$(\ln x)' = \frac{1}{x}$

$$= 9 \times \ln(q+1)\Big|_0^2 - 6$$

$\int \frac{1}{x} = \ln x + C$

$$= 9 \times (\ln(2+1) - \ln(1)) - 6$$

$\int \frac{1}{x+1}\,dx = \ln(x+1) + C$

$$= 9 \times (\ln 3) - 6 \approx 3.89$$

$\int_a^b \frac{1}{x+1}\,dx = \ln(x+1)\Big|_a^b$

$= \ln(b+1) - \ln(a+1)$

$$SP = p^* q^* - \int_0^{q^*} P_O(q)\,dq = 3\times 2 - \int_0^2 (q+1)\,dq$$

$$= 6 - \frac{1}{2}(q+1)^2\Big|_0^2$$

$(x^2)' = 2x$

$(\frac{1}{2}x^2)' = x$

$$= 6 - \frac{1}{2}\left((2+1)^2 - 1\right)$$

$\int x = \frac{1}{2}x^2 + C$

$$= 6 - \frac{1}{2}(9-1) = 6-4 = 2.$$

$\int_a^b x = \frac{1}{2}b^2 - \frac{1}{2}a^2$

$\int_a^b (x+1) = \frac{1}{2}(x+1)^2\Big|_a^b$

$= \frac{1}{2}(b+1)^2 - \frac{1}{2}(a+1)^2$.

$\int_a^b f(x)\,\underset{\Delta x}{dx}$

Ex3.

1. Intégration par parties: Soit u et v deux fonctions sur $[a.b]$.

On a alors $\int_a^b u(x)\,v'(x)\,dx = u(x)\,v(x)\Big|_a^b - \int_a^b u'(x)\,v(x)\,dx$

Δ ⓐ. $\int x^2 \ln(x)\,dx$

ⓑ $\int \underbrace{x}_{\text{poly}}\ \underbrace{e^{2x}}_{\text{exp} \to v'}\ dx.$

petite astuce:

~~A~~ L P E S

fonc Inver .. ← A; L → log; P → x^α; E → Exp; S → Sin cosx

$\xleftarrow{\quad}$ u v

On pose $u(x) = x$ $\quad v' = e^{2x}$, donc $u'(x) = 1$

$v(x) = \frac{1}{2}e^{2x}$

$\frac{1}{2}e^{2x} \times 2 = e^{2x} = v'$

la formule d'intégration par parties:

$$\int \underbrace{x}_{u}\ \underbrace{e^{2x}}_{v'}\,dx = uv - \int u'v\,dx$$

$$= x \times \frac{1}{2}e^{2x} - \int 1 \times \frac{1}{2}e^{2x}\,dx$$

$$= \frac{1}{2}x\,e^{2x} - \frac{1}{2}\int e^{2x}\,dx$$

$$= \frac{1}{2}x\,e^{2x} - \frac{1}{2} \times \left(\frac{1}{2}e^{2x} + c\right)$$

$$= \frac{1}{2}x\,e^{2x} - \frac{1}{4}e^{2x} + C$$

$(e^{2x})' = e^{2x} \times 2$

$\left(\frac{1}{2}e^{2x}\right)' = e^{2x}$

$\int e^{2x} = \frac{1}{2}e^{2x} + c$

$c \times \frac{1}{2} = c'$

$c \times (-\frac{1}{2}) = c'$

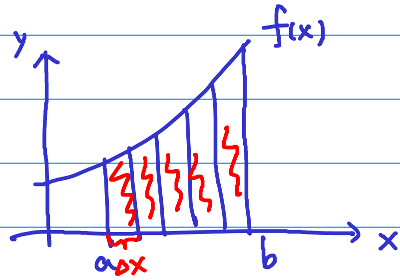
ⓐ $\int \underbrace{x^2}_{\text{poly} \to v'}\ \underbrace{\ln(x)}_{\text{Log} \to u}\ dx$

$u(x) = \ln(x)$ $\quad v' = x^2$

$u'(x) = \frac{1}{x}$, $\quad v = \frac{1}{3}x^3$

$\int_a^b f(x)\,dx.$

$dx \approx \Delta x.$



l'integral $\int_a^b f$ est la limite des surfaces des rectangles.

la formule IPP : $\int_a^b u v' \, dx = uv \Big|_a^b - \int_a^b u' v \, dx$

Ex3.

2. △ ⓐ

ⓑ $\int_0^{+\infty} \underbrace{x}_{u} \, \underbrace{e^{-2x}}_{v'} \, dx$

x^α poly. $\to u$ Ex $\to v'$

On pose $u(x) = x$, $v'(x) = e^{-2x}$.

donc $u'(x) = 1$. $v(x) = -\frac{1}{2} e^{-2x}$

$(e^{-2x})' = e^{-2x} \times (-2)$

$(-\frac{1}{2} e^{-2x})' = e^{-2x}$

$\int e^{-2x} = -\frac{1}{2} e^{-2x} + C$

la formule IPP.

$$\int_0^{+\infty} x e^{-2x} \, dx = uv \Big|_0^{+\infty} - \int_0^{+\infty} u'(x) v(x) \, dx$$

$$= x \times (-\tfrac{1}{2} e^{-2x}) \Big|_0^{+\infty} - \int_0^{+\infty} 1 \times (-\tfrac{1}{2} e^{-2x}) \, dx$$

$$= -\tfrac{1}{2} x e^{-2x} \Big|_0^{+\infty} + \tfrac{1}{2} \int_0^{+\infty} e^{-2x} \, dx$$

$\frac{\infty}{\infty}$

$$= -\tfrac{1}{2} \frac{x}{e^{2x}} \Big|_0^{+\infty} + \tfrac{1}{2} \times (-\tfrac{1}{2} e^{-2x}) \Big|_0^{+\infty}$$

$\lim\limits_{x \to +\infty} \frac{x}{e^{2x}} = 0$

$$= -\tfrac{1}{2} (0 - \tfrac{0}{1}) + \tfrac{1}{2} \times (-\tfrac{1}{2}) \; e^{-2x} \Big|_0^{+\infty}$$

$\frac{1}{e^{2x}}$

$$= -\tfrac{1}{4} \left(e^{-2x} \Big|_0^{+\infty} \right)$$

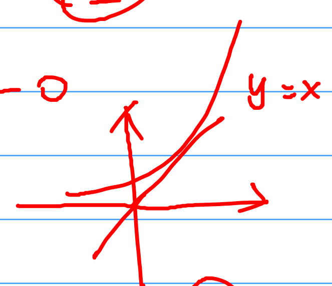
$$= -\tfrac{1}{4} \underbrace{(0 - 1)}_{-1} = -\tfrac{1}{4} \times (-1) = \tfrac{1}{4}.$$

$$0 = \frac{x}{e^{2x}} \Big|_0^{+\infty} = \frac{\infty}{e^{2 \times (+\infty)}} - \frac{0}{e^{2 \times 0}} = \lim_{x \to +\infty} \frac{x}{e^{2x}} - \frac{0}{1} = 0$$

$$\frac{x}{e^{2x}} \Big|_b^a = \frac{a}{e^{2a}} - \frac{b}{e^{2b}}$$

$$= \lim_{x \to +\infty} \frac{1}{e^{2x} \cdot 2} - 0$$

$$= \frac{1}{\infty} = 0$$



$x \to +\infty$

$e^x \to +\infty$

$$e^{-2x} \Big|_0^{+\infty} = \frac{1}{e^{2 \times (+\infty)}} - \frac{1}{e^{2 \times 0}} = 0 - 1$$

$$= -1.$$

Ex4: Changement de variable :

$$\int_a^b f(u)\,du = \int_\alpha^\beta f(g(x))\cdot g'(x)\,dx$$

$$\boxed{a = g(\alpha) \qquad b = g(\beta)}$$

①

② $\int_{-1}^{+1} \frac{x^2}{(2+x^3)^4}\,dx$.

$(x^3)' = 3x^2$

On pose $u = g(x) = x^3$, d'où $g'(x) = 3x^2$. donc on écrit $du = d(g(x))$

$= g'(x)\,dx$

$= 3x^2\,dx$

$\int_{-1}^{1} \longrightarrow \int_{g(-1)}^{g(1)} \rightarrow \int_{-1}^{1}$

Aussi, $\left[\begin{array}{l} x=-1.\ g(-1) = (-1)^3 = -1. \\ x = 1.\ g(1) = 1^3 = 1. \end{array}\right.$

Donc. $\int_{-1}^{+1} \frac{x^2}{(2+x^3)^4}\,dx = \int_{-1}^{1} \frac{1}{3(2+x^3)^4} \underbrace{3x^2\,dx}_{du}$

$= \int_{-1}^{1} \frac{1}{3\times(2+u)^4}\,du$

$= \frac{1}{3}\int_{-1}^{1} \frac{1}{(\underbrace{2+u}_{y})^4}\,du$

$= \frac{1}{3}\times\left(-\frac{1}{3}(2+u)^{-3}\Big|_{-1}^{1}\right)$

$= -\frac{1}{9}\,(2+u)^{-3}\Big|_{-1}^{1}$

$= -\frac{1}{9}\left(\frac{1}{(2+1)^3} - \frac{1}{(2-1)^3}\right)$

$= -\frac{1}{9}\left(\frac{1}{27} - 1\right) = \frac{1}{9}\left(1 - \frac{1}{27}\right)$

$= \frac{1}{9}\times\frac{26}{27} = \frac{26}{9\times 27}$

$(y^{-3})' = -3\times y^{-4}$

$\underline{-\frac{1}{3}(y^{-3})'} = y^{-4}$

$= \frac{1}{y^4}$

$\int \frac{1}{y^4} = -\frac{1}{3}y^{-3} + C$

$\int \frac{1}{(2+u)^4} = -\frac{1}{3}(2+u)^{-3} + \cancel{C}$

Ex 4

④ $\int \frac{1}{x} (\ln x)^3 \, dx$. $(\ln x)' = \frac{1}{x}$

$\underbrace{\ln x}_{u}$

On pose $u = g(x) = \ln x$, donc $du = \frac{1}{x} \, dx$.

$$\int \frac{1}{x} (\ln x)^3 \, dx = \int (\underbrace{\ln x}_{u})^3 \frac{1}{x} \, dx$$

$$= \int u^3 \, du$$

$$= \frac{1}{4} u^4 + C.$$

$$= \frac{1}{4} (\ln x)^4 + C$$

$(u^4)' = 4u^3$

$(\frac{1}{4} u^4)' = u^3$

$\int u^3 du = \frac{1}{4} u^4 + C$

$\square$.